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1977 J. Phys. A: Math. Gen. 10 1225

(http://iopscience.iop.org/0305-4470/10/7/019)

J. Phys. A: Math. Gen., Vol. 10, No. 7, 1977. Printed in Great Britain. © 1977

Lagrange multiplier formalism for a spin-$\frac{3}{2}$ field

K Babu Joseph and M Sabir

Department of Physics, Cochin University, Cochin-682022, India

Received 20 December 1976

Abstract. It is proposed that the method of keeping the subsidiary conditions separate from the equations of motion in a consistent way by means of a Lagrange multiplier may be the solution to the various pathologies afflicting high-spin field theories. The distinct features of this approach are discussed with reference to a massive spin-$\frac{3}{2}$ field. It is shown that in this formulation there is no causality violation for propagation in an external field and the energy spectrum in a homogeneous magnetic field is real. Quantization is carried out in a Hilbert space with indefinite metric. For minimal electromagnetic interaction a unitary S matrix is constructed by introducing a fictitious particle and an additional vertex.

1. Introduction

The problem of formulating a consistent field theory of high-spin particle (spin $s \geqslant 1$) interactions is still an unsolved one. Johnson and Sudarshan (1961) were the first to observe an anomaly in the quantization of a spin-$\frac{3}{2}$ field in the presence of an external electromagnetic field. After the pioneering work of Velo and Zwanziger (1969a, b), which demonstrated acausal propagation in classical versions of interacting high-spin field theories, further investigations by a number of workers have revealed the existence of a variety of maladies afflicting high-spin field theory at the classical as well as the quantized level (Shamaly and Capri 1972, Jenkins 1973a, b, c, d, Singh 1973). Loss of constraints, breakdown of Lorentz covariance, and appearance of imaginary energy eigenvalues are, aside from causality violation, forms of pathologies which haunt high-spin field theories (Velo and Zwanziger 1969b, Velo 1972, Nath *et al* 1971, Jenkins 1974, Babu Joseph and Sabir 1976a, b, Tsai and Yildiz 1971, Mathews and Seetharaman 1973). Recently some suggestions have arisen as to possible ways of curing high-spin field theory of its inherent maladies (Velo 1972, Fukuyama and Yamamoto 1973, Prabhakaran *et al* 1975). In this paper we explore the possibility of eliminating the pathological behaviour of high-spin field theory by introducing a Lagrange multiplier formalism.

In the conventional formulation, the subsidiary conditions required for the elimination of redundant components are derived along with the equations of motion by the variation of a Lagrangian. These constraint relations, as a rule, get modified when interaction terms are introduced into the Lagrangian. We propose that the alternative of keeping the subsidiary conditions separate from the equations of motion in a consistent way by means of a Lagrange multiplier may be a solution to the above mentioned pathologies of high-spin field theories. Nakanishi, Hsu and Sudarshan (Nakanishi 1972, Hsu 1974a, b, Hsu and Sudarshan 1974) have made use of a Lagrange multiplier formalism to construct manifestly renormalizable theories of massive vector fields and massive and massless Yang–Mills fields. But our motivation has a different origin. We try to take into account the existence of constraint relations from the very outset and these constraint relations are kept away from the influence of interactions which otherwise lead to troubles of various sorts.

The distinct features of the method outlined above are illustrated below with reference to a spin-$\frac{3}{2}$ field. In § 2 we set up the Lagrange multiplier formalism for a massive spin-$\frac{3}{2}$ field. Quantization of the field is carried out in § 3. In § 4 we demonstrate the absence of the usual pathologies in the suggested framework when the interaction with an electromagnetic field is introduced. Then § 5 deals with the question of unitarity of the S matrix in this formalism, and § 6 sums up the advantages and limitations of the present approach.

2. Formulation

We choose to describe a spin-$\frac{3}{2}$ field by means of a 16-component vector-spinor ψ_μ (spinor index suppressed). The most general Lagrangian leading to an equation of motion that gives up to first derivatives has the form

$$\mathscr{L} = \psi_\mu(x)\Lambda_{\mu\nu}\psi_\nu(x) \tag{1}$$

with

$$\Lambda_{\mu\nu} = -(\gamma\partial + m)\delta_{\mu\nu} - A(\gamma_\mu\partial_\nu + \gamma_\nu\partial_\mu) - B\gamma_\mu\gamma_\lambda\partial_\lambda\gamma_\nu - Cm\gamma_\mu\gamma_\nu. \tag{2}$$

This Lagrangian with the conditions

$$A \neq -\tfrac{1}{2} \qquad B = \tfrac{3}{2}A^2 + A + \tfrac{1}{2} \qquad C = -(3A^2 + 3A + 1) \tag{3}$$

yields the Rarita–Schwinger formulation for the irreducible spin-$\frac{3}{2}$ field which is known to be afflicted by troubles of different sorts when minimal electromagnetic interaction is introduced.

In the present formulation we take (1) as the Lagrangian to start with. We now assume that there is a constraint relation between the 16 components of the field of the form

$$\gamma_\mu\psi_\mu(x) = 0. \tag{4}$$

Unlike in the usual treatment, the Lagrangian (1) is varied under the assumption that the constraint relation (4) exists independently and it is incorporated into the Lagrangian by means of a Lagrange multiplier. The Lagrange multiplier thus introduced must be a function of space–time and hence may be treated as an additional field variable. The variation of the multiplier field in the Lagrangian yields the constraint relation *a posteriori*. The effective Lagrangian with the constraint incorporated is

$$\mathscr{L} = \bar{\psi}_\mu\Lambda_{\mu\nu}\psi_\nu + \eta\bar{\xi}\gamma_\mu\psi_\mu + \eta\bar{\psi}_\mu\gamma_\mu\xi \tag{5}$$

where the multiplier ξ must be a four-spinor for the Lagrangian to be a Lorentz-invariant and η is a real number. The resulting equations of motion are given by:

$$[(\gamma\partial + m)\delta_{\mu\lambda} + A(\gamma_\mu\partial_\lambda + \gamma_\lambda\partial_\mu) + B\gamma_\mu\gamma\partial\gamma_\lambda + Cm\gamma_\mu\gamma_\lambda]\psi_\lambda = \eta\gamma_\mu\xi \tag{6}$$

as well as equation (4). Because of equation (4), equation (6) reduces to

$$(\gamma\partial + m)\psi_\mu + A\gamma_\mu\partial_\lambda\psi_\lambda = \eta\gamma_\mu\xi. \tag{7}$$

Multiplying equation (7) successively with γ_μ and ∂_μ and making use of (4) we obtain the relations

$$(\tfrac{1}{2}+A)\partial_\lambda\psi_\lambda=\eta\xi \tag{8}$$

$$m\partial_\lambda\psi_\lambda+(1+A)\gamma\partial\partial_\lambda\psi_\lambda=\eta\gamma\partial\xi. \tag{9}$$

When $A\neq-\frac{1}{2}$ we substitute equation (8) into equation (9) to derive

$$(\gamma\partial+M)\xi=(\gamma\partial+M)\partial_\lambda\psi_\lambda=0 \tag{10}$$

where

$$M=\frac{m}{1+A-\frac{1}{2}\eta+A\eta}.$$

When $A=-\frac{1}{2}$, we have, from equation (8) $\xi=0$ but from equation (9) we see that we can still derive the relation

$$(\gamma\partial+M)\partial_\lambda\psi_\lambda=0.$$

We therefore, have in our theory in addition to the spin-$\frac{3}{2}$ particle of mass m a spin-$\frac{1}{2}$ particle of mass M satisfying the Dirac equation. This becomes further evident from an inspection of the equations of motion in the rest frame as is done below. In the following, for the sake of convenience, we let $\eta=1$, so that $M=2m$.

From equations (7), (8) and (9) we find

$$(\gamma\partial+m)\psi_\mu=\tfrac{1}{2}\gamma_\mu\partial_\lambda\psi_\lambda=\gamma_\mu\xi/(1+2A) \tag{11}$$

where the last equality holds if $A\neq-\frac{1}{2}$. The components of ψ_μ do not satisfy the Klein–Gordon equation, but equations (10) and (11) together yield

$$(\square-4m^2)(\gamma\partial+m)\psi_\mu=0 \tag{12}$$

and

$$(\square-4m^2)(\square-m^2)\psi_\mu=0. \tag{13}$$

From equation (11) and its conjugate, with the aid of equation (9), we can derive the conserved current

$$j_\mu=\mathrm{i}\bar{\psi}_\nu\gamma_\mu\psi_\nu. \tag{14}$$

The total charge is given by

$$Q=\int \mathrm{d}^3x\ (\psi_1^\dagger\psi_1+\psi_2^\dagger\psi_2+\psi_3^\dagger\psi_3-\psi_0^\dagger\psi_0^\dagger). \tag{15}$$

In the present formulation the total charge is not positive definite. This fact may be demonstrated by examining equation (11) in the rest frame. Let $\psi_\lambda\sim w_\lambda(\boldsymbol{k},E)\,\mathrm{e}^{\mathrm{i}(\boldsymbol{k}.\boldsymbol{x}-Et)}$ be the solution of equation (11) with positive energy. In the rest system with $\boldsymbol{k}=0$ equation (11) becomes

$$(-\gamma_4E+m)w_4=-\tfrac{1}{2}\gamma_\mu Ew_4. \tag{16}$$

For $\mu=4$ we get

$$(-\tfrac{1}{2}\gamma_4E+m)w_4=0. \tag{17}$$

If $E=m$, w_4 will vanish identically as is the case in the Rarita–Schwinger theory.

However, our formulation also admits the value $E=2m$, so w_4 has non-vanishing components corresponding to this. The non-vanishing terms contribute negative terms in the expression (15) and the total charge becomes indefinite.

Before turning to the problem of quantization we take a closer look at equation (16) with a view to identifying the different spin components. With $\mu=i=1,2,3$ and $E=m$ equation (16) becomes

$$(-\gamma_4+1)w_i=0. \tag{18}$$

This implies that only the lower components of w_4 survive and these six components are reduced to four independent components by the constraint relation which now assumes the form

$$\boldsymbol{\sigma}\cdot\boldsymbol{v}=o \tag{19}$$

where v_i is a two-component spinor. However, when $E=2m$ equation (16) becomes

$$(-2\gamma_4+1)w_i=-\tfrac{1}{2}\gamma_i w_4. \tag{20}$$

We may use this relation to express the non-vanishing components of w_i in terms of w_4 so that there are only two independent solutions corresponding to the value $E=2m$, which together constitute the spin-$\frac{1}{2}$ part of the field.

3. Quantization

Since on the Lagrangian (5) ψ_μ are all varied independently, we can use the canonical quantization method to derive the commutation relations of field components. Canonically conjugate momenta π_μ of ψ_μ are

$$\pi_\mu=\mathrm{i}\bar{\psi}_\nu(L_4)_{\nu\mu} \tag{21}$$

where $(L_4)_{\mu\nu}=\gamma_4\delta_{\mu\nu}+A(\gamma_\mu\delta_{\nu4}+\gamma_\nu\delta_{\mu4})+B\gamma_\mu\gamma_4\gamma_\nu$. Standard canonical commutation relations are given by

$$\{\psi_\mu(x),\pi_\nu(x')\}_{x_0=x_0'}=\mathrm{i}\delta_{\mu\nu}\delta(\boldsymbol{x}-\boldsymbol{x}') \tag{22}$$

from which it follows

$$\{\psi_\mu(x),\bar{\psi}_\nu(x')\}_{x_0=x_0'}=(L_4)^{-1}_{\mu\nu}\delta(\boldsymbol{x}-\boldsymbol{x}'). \tag{23}$$

In the Rarita–Schwinger theory with the restriction (3) L_4 is singular and its inverse does not exist. When the conditions (3) are not satisfied $(L_4)^{-1}$ is given by

$$\begin{aligned}(L_4)^{-1}_{\mu\nu}=[&(1+2A+3A^2-2B)\gamma_4\delta_{\mu\nu}-(A^2-A-2B)(\gamma_\mu\delta_{\nu4}+\gamma_\nu\delta_{\mu4})\\&-2(2A+A^2+2B)\gamma_4\delta_{\mu4}\delta_{\nu4}\\&+(A^2-B)\gamma_\mu\gamma_4\gamma_\nu]/(1+2A+3A^2-2B).\end{aligned} \tag{24}$$

Since the parameters A and B are arbitrary we have the freedom to choose them in such a way that the commutation relation (23) is consistent with the subsidiary condition (4). It is easy to verify that consistency is achieved if we choose $A=-1$ and $B=\frac{3}{4}$. With this choice the commutation relations may be written as

$$\{\psi_\mu(x),\bar{\psi}_\nu(x')\}_{x_0=x_0'}=[\delta_{\mu\nu}\gamma_4+\tfrac{1}{2}\gamma_\mu\gamma_4\gamma_\nu-(\delta_{\mu4}\gamma_\nu+\delta_{\nu4}\gamma_\mu)+2\gamma_4\delta_{\mu4}\delta_{\nu4}]\delta(\boldsymbol{x}-\boldsymbol{x}'). \tag{25}$$

The requirements of relativistic covariance, locality and equations (12) and (13) determine the general form of the commutator for arbitrary separation to be

$$\begin{aligned}\{\psi_\mu(x), \bar{\psi}_\nu(x')\} &= -a\mathrm{i}(\gamma\partial - m)[\delta_{\mu\nu} - \tfrac{1}{3}\gamma_\mu\gamma_\nu + \tfrac{1}{3}(\gamma_\mu\partial_\nu - \gamma_\nu\partial_\mu) - (2/3m^2)\partial_\mu\partial_\nu]\Delta(x-x':m) \\ &\quad + b\mathrm{i}(\partial_\mu + \tfrac{1}{2}m\gamma_\mu)(\gamma\partial - 2m)(\partial_\nu + \tfrac{1}{2}m\gamma_\nu)\Delta(x-x':2m). \qquad (26)\end{aligned}$$

In order that the above expression be consistent with the equal time commutation relations (25) we must take $a = 1$ and $b = 2/3m^2$. We can use equations (26) and (8) with $A = -1$ to derive the further commutation relations

$$\begin{aligned}\{\xi(x), \bar{\psi}_\nu(x')\} &= -3\mathrm{i}(\gamma\partial - 2m)(\partial_\nu + \tfrac{1}{2}m\gamma_\nu)\Delta(x-x':2m) \\ \{\xi(x), \bar{\xi}(x')\} &= \frac{27\mathrm{i}m^2}{2}(\gamma\partial - 2m)\Delta(x-x':2m).\end{aligned} \qquad (27)$$

The last of the commutation relations shows that the spin-$\frac{1}{2}$ field has a negative metric in Hilbert space.

The Feynman propagator of the field, defined as the vacuum expectation value of the time-ordered product, is easily evaluated in the usual manner. In the present case the normal-dependent terms cancel each other, and we rigorously have

$$\begin{aligned}S_{\mathrm{F}\mu\nu}(x_1 - x_2) &= \langle 0|T\psi_\mu(x_1)\bar{\psi}_\nu(x_2)|0\rangle \\ &= -(\gamma\partial - m)[\delta_{\mu\nu} - \tfrac{1}{3}\gamma_\mu\gamma_\nu + \tfrac{1}{3}(\gamma_\mu\partial_\nu - \gamma_\nu\partial_\mu) - (2/3m^2)\partial_\mu\partial_\nu]\Delta_\mathrm{F}(x_1 - x_2:m) \\ &\quad - (2/3m^2)(\partial_\mu + \tfrac{1}{2}m\gamma_\mu)(\gamma\partial - 2m)(\partial_\nu + \tfrac{1}{2}m\gamma_\nu)\Delta_\mathrm{F}(x_1 - x_2:2m). \qquad (28)\end{aligned}$$

4. Interaction and pathologies

We now discuss the interaction of the above described mixed spin-$\frac{3}{2}$–spin-$\frac{1}{2}$ field with an electromagnetic field. To begin with, we consider the coupling to an external field.

4.1. Causality of propagation

Introducing minimal coupling into the Lagrangian (5) with the subsidiary condition unchanged we derive the gauge invariant field equation in the presence of an external electromagnetic field as

$$(\gamma\pi + m)\psi_\mu + A\gamma_\mu\pi_\lambda\psi_\lambda = \gamma_\mu\xi. \qquad (29)$$

Taking into account the subsidiary condition (4) it is easily verified that ξ satisfies the equation

$$(\gamma\pi + 2m)\xi = \frac{4\mathrm{i}e}{1+2A}F_{\mu\lambda}\gamma_\lambda\psi_\mu. \qquad (30)$$

Expressing ξ in terms of $\pi_\lambda\psi_\lambda$ we can rewrite (29) in the form

$$(\gamma\pi + m)\psi_\mu - \tfrac{1}{2}\gamma_\mu\pi_\lambda\psi_\lambda = 0. \qquad (31)$$

It is evident from this equation that the propagation character is unaffected by the coupling and that the characteristic determinant $D(n)$ where n denotes a unit vector normal to the wavefront, has the same value as in the free field case and is given by

$$D(n)=\tfrac{1}{2}(n^2)^8 \tag{32}$$

so that the propagation is light-like, and hence causal.

4.2. *Eigenvalues in a homogeneous magnetic field*

The method developed by Mathews (1974) is applied to determine the eigenvalues of the spin-$\frac{3}{2}$ particle in a homogeneous magnetic field. Define

$$\pi_\pm=\pi_1\pm \mathrm{i}\pi_2, \qquad \psi_\pm=\psi_1\pm \mathrm{i}\psi_2, \qquad \gamma_\pm=\tfrac{1}{2}(\gamma_1\pm \mathrm{i}\gamma_2).$$

Equation (31) with $\mu=1,2$ can be manipulated to give two equations that involve ψ_+ and ψ_- only decoupled from each other

$$E\gamma_4\gamma_+\psi_+ +2(\gamma_+\pi_+ +\gamma_-\pi_-)\gamma_-\psi_+ -(\gamma_3\pi_3-m)\gamma_+\psi_+=0 \tag{33}$$

$$E\gamma_4\gamma_-\psi_- +2(\gamma_+\pi_+ +\gamma_-\pi_-)\gamma_+\psi_- -(\gamma_3\pi_3-m)\gamma_-\psi_-=0. \tag{34}$$

With ψ_+ partitioned in the form $\binom{\phi_+}{\chi_+}$, equation (33) becomes the pair of coupled equations

$$(E+m)\sigma_+\chi_+ +(2eH)^{1/2}(2a\sigma_+\sigma_- +a_3\sigma_3\sigma_+)\phi_+=0 \tag{35}$$

$$(E-m)\sigma_+\phi_+ +(2eH)^{1/2}(2a\sigma_+\sigma_- +a_3\sigma_3\sigma_+)\chi_+=0 \tag{36}$$

where $a=\mathrm{i}(2eH)^{1/2}\pi_+$, $a^\dagger=\mathrm{i}(2eH)^{1/2}\pi_-$. The solutions of equation (35) are of the general form

$$\phi_+=c_1|n_1\beta\rangle \qquad \chi_+=c_2|n_2\beta\rangle. \tag{37}$$

Substituting in (35) and equating the coefficients of $|n\alpha\rangle$ on both sides we obtain relations connecting c_1 and c_2 which can be written compactly as

$$Hc=Ec \tag{38}$$

where

$$c=\begin{pmatrix}c_1\\c_2\end{pmatrix} \quad \text{and} \quad H=\begin{pmatrix} m & -(2eH)^{1/2}a_3\\ -(2eH)^{1/2}a_3 & -m\end{pmatrix}.$$

The eigenvalues of H are readily found to be given by

$$E^2=m^2[1+(2eH/m^2)a_3^2] \tag{39}$$

The same procedure as above can be applied to equation (34). The components ψ_3 and ψ_4, on the other hand, can be expressed in terms of ψ_+, ψ_- and their space-derivatives.

5. *S* matrix and unitarity

In this section we examine the properties of the S matrix for the interaction of our spin-$\frac{3}{2}$ field with a quantized electromagnetic field with the Lagrangian

$$\mathscr{L}=\bar{\psi}_\mu\Lambda_{\mu\nu}\psi_\nu+\bar{\xi}\gamma_\mu\psi_\mu+\bar{\psi}_\mu\gamma_\mu\xi-\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} \tag{40}$$

where the conjugation is now taken to be that in an indefinite metric space.

Physical states are defined by the condition

$$\xi^{(+)}(x)|\text{physical}\rangle = 0 \qquad \bar{\xi}^{(+)}(x)|\text{physical}\rangle = 0. \tag{41}$$

Because of the interaction of the negative metric particle, as is evident from equation (30), the S matrix defined in the physical subspace of the indefinite metric space will not be automatically unitary. However, following Hsu (1974a, b) we can restore the unitarity of the S matrix by introducing a fictitious particle and a new vertex. To show this we consider the scattering amplitude for the self-energy process $\psi(p) \rightarrow \psi(p-k)\gamma(k) \rightarrow \psi(p)$ which is obtained as

$$S^{(2)} = \frac{e^2}{V}\int d^4k\, \bar{u}^{\alpha}_{\mu}(p)\gamma_{\lambda}\Big(d_{\mu\nu}\,(ik)\frac{-i}{(p-k)^2+m^2} + \frac{2}{3m^2}[i(p-k)_{\mu} + \tfrac{1}{2}m\gamma_{\mu}]$$
$$\times \frac{-1}{\gamma(p-k)-2im}[i(p-k)_{\nu} + \tfrac{1}{2}m\gamma_{\nu}]\gamma_{\rho}u^{\alpha}_{\nu}(p)\delta_{\lambda\rho}\frac{-i}{k^2}\Big) \tag{42}$$

where

$$d_{\mu\nu}(\partial) = -(\gamma\partial - m)[\delta_{\mu\nu} - \tfrac{1}{3}\gamma_{\mu}\gamma_{\nu} + \tfrac{1}{3}(\gamma_{\mu}\partial_{\nu} - \gamma_{\nu}\partial_{\mu}) - (2/3m^2)\partial_{\mu}\partial_{\nu}].$$

This contains an extra amplitude coming from

$$\frac{e^2}{V}\int d^4k\, \frac{2}{3m^2}\bar{u}^{\alpha}_{\mu}(p)\gamma_{\lambda}[i(p-k)_{\mu} + \tfrac{1}{2}m\gamma_{\mu}]\frac{-1}{\gamma(p-k)-2im}[i(p-k)_{\nu}$$
$$+ \tfrac{1}{2}m\gamma_{\nu}]\gamma_{\rho}u^{\alpha}_{\nu}(p)\delta_{\lambda\rho}\frac{-i}{k^2}.$$

To remove this extra amplitude we introduce a fictitious spin-$\frac{1}{2}$ particle F of mass $2m$ into the theory. If the interaction of the F particle has the form $\bar{\psi}_{\mu}\gamma_{\nu}(\partial_{\nu} + \frac{1}{2}m\gamma_{\nu})FA_{\mu}$ it is not difficult to verify that the extra amplitude is cancelled (in the second order) by the contribution from the process $\psi(p) \rightarrow F(p-k)\gamma(k) \rightarrow \psi(p)$. It is conjectured that this cancellation is valid in all orders of perturbation theory.

6. Conclusion

The Lagrange multiplier formalism, though successful in eliminating some of the pathologies such as causality violation and existence of imaginary energy eigenvalues in homogeneous magnetic fields and the difficulties of quantization, leads to a multi-mass, multi-spin formalism with indefinite metric. This result seems to be in accord with the conjecture of Prabhakaran *et al* (1975) that for half-integer spins greater than $\frac{1}{2}$ causality in the presence of electromagnetic interaction may be retained only if we start with a free theory which is reducible, and wherein the total charge is indefinite. The notable fact is that the requirement that the subsidiary conditions remain separate leads naturally to such a formalism. Unlike in the Rarita–Schwinger case, the Feynman propagator turns out to be covariant in the present theory of a spin-$\frac{3}{2}$ field. The results of this paper lend ample support to the currently popular view that if there exist fundamental high-spin fields, they must have a multi-mass multi-spin structure. Though an indefinite metric is inescapable in this approach, introduction of a further ghost field in the context of electromagnetic coupling serves to preserve unitarity of the theory.

Acknowledgments

We thank Professor E C G Sudarshan for several valuable comments on the content of this paper. One of us (MS) is grateful to the University Grants Commission, New Delhi, for the award of a Junior Research Fellowship.

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